**Research article**

# **DEVELOPMENT OF MATHEMATICAL MODEL TO MONITOR THE EFFECT OF EFFLORESCENCE IN CONCRETE STRUCTURE**

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# **Abstract**

Generation of efflorescence in concrete formation are through several sources, one of the major ways are through the exposure of concrete to water subjecting it to leaching and deposition of minerals. These sources are the major way leaching are develop, such related effect produces dissolution and precipitation of minerals in water from any sources, this also include ground water to surface water or water contained in or transported by structures, leaching are through chemical dissolutions including removal of water soluble in concrete, these are some conditions that are associated with efflorescence to cause concrete deformation, study of these type of concrete deformation state that Efflorescence refers to a white, powdery to hard deposit commonly calcium carbonate or sodium sulfate hydrate, but may consist of other salt such as formed when water containing dissolved minerals migrates through concrete and subsequently evaporate on exposed to surface, the rate of these type of deformation were evaluated, these condition were expressed through mathematical expressions, the development of its governing equation generated model that can determine the rate efflorescence deformation of concrete formation. **Copyright © WJBASR, all rights reserved.** 

**Keywords:** mathematical model, efflorescence, and concrete structure

# **1. Introduction**

Although some models [1, 2, 4, and 11] have been proposed in the past that consider fundamental mechanisms responsible for deterioration, little work has been performed to understand how these mechanisms interact [9]. For

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example, the interaction between freeze/thaw deterioration and fatigue cracking can result in cumulative damage that may act to accelerate deterioration of pavements or bridge decks. Micro-cracks due to mechanical loading may permit the ingress of excessive water, thereby increasing the susceptibility to freeze/thaw damage. Recent studies [Krauss et al., 1995] have shown that even small cracks (0.05mm) can act to increase the permeability of the system and thereby accelerate deterioration [11]. The chain reaction that can occur as a result of the interaction between damage caused by several sources is anticipated to be responsible for a more accelerated deterioration of the concrete structures than what would be generated by any one mechanism independently. Durability parameters measured using well-cured undamaged specimens may be misleading and may overestimate performance [3,9]. For example, recent investigations [3] showed that surface cracking in concrete can substantially increase the surface permeability even though these micro-cracks do not have a substantial influence on the mechanical behaviour of concrete. In a similar study,[9] observed that fine, distributed micro-cracks permitted water penetration to a depth of 30mm. They also observed that water and deicing salts could penetrate these cracks and thereby cause a significant weight change during freeze and thaw cycling. Similarly, condition [9] showed that during repeated freezing and thawing cycles, water absorption would become accelerated as a result of micro-cracking, thereby accelerating freeze and thaw damage, In the Midwest of US, D-cracking is due to the destruction of the aggregate by freezing and thawing [6]: "D-cracking is a form of Portland cement concrete deterioration associated primarily with the use of coarse aggregates that disintegrate when they become saturated and are subject to repeated cycles of freezing and thawing. Also curling and warping would induce stress concentrations at corners and edges of the concrete slab, which make it more susceptible to D-cracking [7]. Surface scaling of concrete has been aggravated by the widespread use of de-icing salts. The areas showing the greatest deterioration are usually where pending of water and salt solutions or continuous wetting has occurred during repeated cycles of freezing and thawing [1,5and 8]. If the concrete is fragmented and casts of ice crystals are abundant on almost all of the fragments, this indicates that the concrete was forced apart by the expansion of the ice formed before the paste achieved final setting [8]. However, the distribution of air is of outmost importance to the freezing and thawing resistance. The size and distribution of air voids can easily be observed using epoxy impregnation [5].

### **2. Theoretical background**

Concrete expose to water is subjected to leaching and disposition of mineral deposit, leaching and deposition are related effect that is produced by dissolution and precipitation of water from any sources, including ground water, surface water or water contained or transport by concrete structure, leaching refer to chemical dissolution and removal of water soluble constituents in concrete. The deposition of mineral deposit includes both scaling or the formation of incrustation and efflorescence's scale deposit that are generally hard, layered precipitate of calcium sulphate, calcium carbonate or magnesium salt such as produced by the heating of evaporation of water containing dissolved salt. Efflorescence refers to a white, powdery to hard deposit commonly calcium carbonate or sodium sulfate hydrate, but may consist of other salt such as formed when water containing dissolved minerals migrates through concrete and subsequently evaporate on exposed surface, minerals deposits may form on exposed surface where water seeps through concrete along joints and cracks of concrete structures. Such as dam face, pipelines and

tunnels, retaining walls and abutments and building foundations, occasionally surface evaporation leads to transport of water containing various ion through permeable concrete, leading to deposition of salts on horizontal slabs or stem wall close to the soil line, these kind of deposit may be objectionably for several reason, they are unsightly and may adversely affect the performance of a structure by reducing the effectiveness of heat exchanger surface or by obstructing the passage of water in drain holes and water tunnels [Joseph and James,2006].

# **3. Governing Equation**





By direct integration

$$
LnT = \frac{-\lambda^2}{D}x\tag{11}
$$

$$
\Rightarrow T = D\ell^{\frac{\lambda^2}{D}t} \tag{12}
$$

Combining (9) and (12) yields

$$
C(Z,t) = \left( ACos \frac{\lambda}{\sqrt{J}} t + B Sin \frac{\lambda^2}{\sqrt{D}} Z \right) D \ell^{\frac{-\lambda^2}{D}t}
$$
 (13)

At  $t = 0$   $C(o) = Co$ 

$$
Co = ACos \frac{\lambda}{\sqrt{J}} t + B Sin \frac{\lambda}{\sqrt{D}} Z \frac{\partial c}{\partial Z} \bigg|_{Z=0, B} = 0 \qquad \qquad \dots \dots \dots \dots \dots \tag{14}
$$

From equation (13)

$$
\frac{\partial c}{\partial t} = \left( -A \frac{\lambda}{\sqrt{J}} \sin \frac{\lambda}{\sqrt{J}} t + B \frac{\lambda}{\sqrt{J}} \cos \frac{\lambda}{\sqrt{J}} t \right) D \ell^{\frac{-\lambda^2}{D^2}} \qquad \qquad (15)
$$

At  $Z = 0$ 

$$
0 = \frac{B\lambda}{\sqrt{D}} \ D\ell^{\frac{-\lambda}{D}t} \Rightarrow B = 0 \ D \neq 0
$$
 (16)

$$
C = \left( A \cos \frac{\lambda}{\sqrt{D}} t \right) D \ell^{\frac{-\lambda^2}{D^2}} \tag{17}
$$

$$
C = AD\cos\frac{\lambda}{\sqrt{D}}t\,\ell^{\frac{-\lambda^2}{D}Z} \tag{18}
$$

$$
\frac{\partial c}{\partial t} = AD \frac{\lambda}{\sqrt{J}} \sin \frac{\lambda}{\sqrt{J}} t \ell^{\frac{-\lambda^2}{D}Z} \tag{19}
$$

At 
$$
t = d \frac{\partial c}{\partial t} = 0
$$
 (20)

$$
0 = -AD\frac{\lambda}{\sqrt{J}}\sin\frac{\lambda d}{\sqrt{J}} = n\pi = \frac{\lambda d}{\sqrt{J}}, n = 01, 2
$$
 (21)

$$
\Rightarrow \lambda = n\pi \frac{\sqrt{J}}{d} \tag{22}
$$

So that we have:

$$
C(t, Z) = ADCos = \frac{n\pi\sqrt{J}}{d\sqrt{J}}t e^{\frac{n^2\pi^2J}{d^2D}t}
$$
\n(23)

$$
AD\;Cos\; = \frac{n\pi}{d}t\; \ell^{\frac{n^2\pi^2J}{d^2D}t} \tag{24}
$$

Hence  $AD = Co$ 

$$
C(t, Z) = C_0 \; \ell^{\frac{-n^2 \pi^2 J}{d^2 D}} \; Cos \frac{n \pi}{d} Z \qquad (25)
$$

The expression on the rate efflorescence in concrete structure has been thoroughly evaluated, the developed model has detailed the parameters that causes efflorescence in the concrete formation, base on these condition the expression on Efflorescence refers to a white, powdery to hard deposit commonly calcium carbonate or sodium sulfate hydrate, but may consist of other salt such as formed when water containing dissolved minerals migrates through concrete and subsequently evaporate on exposed surface, minerals deposits may form on exposed surface where water seeps through concrete along joints and cracks of concrete structures. Such as dam face, pipelines and tunnels, retaining walls and abutments and building foundations, occasionally surface evaporation leads to transport of water containing various ions through permeable concrete the developed model assess these conditions on the derive solution that generated the model that will monitor efflorescence from these dimensions.

#### **4. Conclusion**

The effectiveness of the developed model are base on monitoring the rate deformation of concrete structure through the deposition of efflorescence, several researcher has study different ways that causes these type of deformation in concrete structure, there concepts may have worked depending on the developed preventive measures generated from their studies, the development of mathematical model are through thorough evaluation of several mineral behaviours that developed this type of concrete deformation, therefore results from other studies may have generated better solution, the development of this conceptual setting by applying mathematical concept should produces better result, because the causes of these type of concrete deformation has been critically assessed generating the governing equation from various parameters, experts will find this concept more comfortable in application , because of its fast results generation.

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